Homogenous Alternative Semantics

Fabrizio Cariani[1] and Simon Goldstein[2]

[1] Northwestern University
[2] Lingnan University

Abstract

We explore the interaction between conditional excluded middle and simplification of disjunctive antecedents. After showing these principles to be nearly incompatible, we develop an approach that fits in the narrow space they leave open.

1 Introduction

David Lewis's logic for the counterfactual conditional [19] famously invalidates two plausible-sounding principles: simplification of disjunctive antecedents (SDA),[1] and conditional excluded middle (CEM).[2] Simplification is the entailment: $(\mathsf{A} \text{ or } \mathsf{B}) > \mathsf{C} \vdash (\mathsf{A} > \mathsf{C})\ \&\ (\mathsf{B} > \mathsf{C})$. For instance, given SDA, (1) entails (2).

(1) If Hiro or Ezra had come, we would have solved the puzzle.

(2) If Hiro had come, we would have solved the puzzle and if Ezra had come, we would have solved the puzzle.

As for CEM, it is the validity claim: $\vdash (\mathsf{A} > \mathsf{B}) \vee (\mathsf{A} > \neg\mathsf{B})$. A distinctive consequence of CEM is that the negation of $\mathsf{A} > \mathsf{C}$ entails $\mathsf{A} > \neg\mathsf{C}$. For instance, (3) entails (4).

(3) It is not the case that if Hiro had come we would have solved the puzzle.

(4) If Hiro had come, we would not have solved the puzzle.

Much attention has been devoted to these heretical principles in isolation, but relatively little work has considered their interaction. Since there are strong arguments for both principles, it is urgent to investigate how they might be made to fit.

Our pessimistic finding is that the heresies do not mix easily. We present a battery of incompatibility results showing that no traditional theory of conditionals or disjunction can allow them to coexist. Despite these negative findings, we argue that the project of combining CEM and SDA is not hopeless—provided that we are willing to incorporate insights from the linguistics literature within our framework for conditional logic. To validate both principles, we synthesize two tools that can be used to validate each principle individually: the alternative sensitive analysis of disjunction [1] and the theory of homogeneity presuppositions [11].

The resulting theory requires one last heresy: the entailment relation must be intransitive. In particular, while CEM is valid, other principles are invalid that are logical consequences of it.

2 The Case for the Heresies

The main argument for SDA seems to consist entirely in the observation that instances like the one from (1) to (2) sound extremely compelling (see [9, p.453-454]). Obviously, this is

[1] See [9], [10]; [21], [22]; [20].
[2] See [27], [11]; [30], and [16].

not a full defense of SDA, but it creates a strong presumption in its favor—one that would require substantial theoretical argument to be overthrown. Indeed, contemporary approaches in truth-maker semantics (e.g., [10]) are designed around the desire to validate it.

CEM is not typically justified by this direct method. Instead, its defenders propose that various phenomena fall into their proper place if we accept CEM's validity. For example, the inference from (3) to (4) turns out to be an application of disjunctive syllogism. More generally, conditionals with *will* and *would* consequents fail to enter into the scope relations that would be expected if CEM failed [27, p.137-139]. A recent version of this argument relies on data involving attitude verbs that lexicalize negation (see [5]).

(5) I doubt that if you had slept in, you would have passed.

(6) I believe that if you had slept in, you would have failed.

The equivalence is easily explained if CEM is valid (and assuming that failing equals not passing). The speaker doubts *sleep* $>$ *pass*; if there was a way for this conditional to be false other than by *sleep* $>$ *fail* being true, it should be possible to accept (5) without accepting (6). By contrast, it is hard, if not impossible, to explain without CEM. This argument streamlines an older argument for CEM involving the interaction between conditionals and quantifiers.[3] Consider:

(7) No student will succeed if he goofs off.

(8) Every student will fail if he goofs off.

(7) and (8) are intuitively equivalent. They appear to involve quantifiers taking scope over conditionals. Given CEM and this scope assumption, they are predicted equivalent. Take an arbitrary student, and suppose it is false of him that he will succeed if he goofs off. By CEM it follows that he will fail if he goofs off. On reflection, then, the interaction of conditionals and quantifiers also favors the validity of CEM.

Our final argument for CEM is based on the interaction between *if* and *only*.[4] CEM can help explain why *only if* conditionals imply their converses. Consider the following conditionals:

(9) The flag flies only if the Queen is home.

(10) If the flag flies, then the Queen is home.

(11) The flag flies if the Queen isn't home.

(9) entails (10). In [11] this entailment is derived compositionally, on the assumption that *only* in (9) takes wide scope to the conditional. *Only* then negates the alternatives to the conditional *the flag flies if the Queen is home*, which are assumed to include (11). Given some background assumptions, Conditional Excluded Middle and the negation of (11) imply (10).

3 Incompatibility Results

Having introduced our favorite conditional heresies, we show that they are in tension with each other. In keeping with a distinction we have drawn in the previous section, we appeal to two distinct notions of disjunction: (i) natural language *or* and (ii) Boolean disjunction, '$\vee$'. Given the asymmetry we highlighted in how SDA and CEM are justified, it will strengthen

[3] See [15]; [14]; [18]; and [16] for discussion.

[4] See [2] and [11] for discussion.

our argument to refrain from assuming that these have the same meaning. Our results require classical assumptions about the logic of '$\vee$' but very few assumptions about the meaning of *or*.

3.1 Collapse

CEM and SDA together imply collapse to the material conditional, given relatively modest logical assumptions. We assume standard sequent rules for classical connectives as well as the standard structural rules governing classical logic.[5] Among the structural rules, the transitivity of entailment will play a very important role in our discussion. Transitivity follows from Cut when X and Y are empty.

Cut. if $X \vdash \mathsf{A}$ and $Y, \mathsf{A} \vdash \mathsf{B}$, then $X, Y \vdash \mathsf{B}$

Several of our proofs rely on disjunction rules, so it is worth stating them explicitly

Cases. if $X, \mathsf{A} \vdash \mathsf{C}$ and $Y, \mathsf{B} \vdash \mathsf{C}$, then $X, Y, (\mathsf{A} \vee \mathsf{B}) \vdash \mathsf{C}$

$\vee$-Intro. if $X, \mathsf{A} \vdash \mathsf{B}$, $X, \mathsf{A} \vdash \mathsf{B} \vee \mathsf{C}$

To these, add specific assumptions about conditionals (three axioms and one rule). The axioms are *modus ponens* ($\mathsf{A}, \mathsf{A} > \mathsf{C} \vdash \mathsf{C}$), *reflexivity* ($\vdash \mathsf{A} > \mathsf{A}$) and *agglomeration* ($\mathsf{A} > \mathsf{B}, \mathsf{A} > \mathsf{C} \vdash \mathsf{A} > (\mathsf{B} \,\&\, \mathsf{C})$). As for the rule, it is:

Upper Monotonicity. if $\mathsf{B} \vdash \mathsf{C}$, then $\mathsf{A} > \mathsf{B} \vdash \mathsf{A} > \mathsf{C}$

While these assumptions are not entirely uncontroversial, they are generally accepted in the literature. For ease of reference, we call this combination of assumptions *the classical package*.

We can now state our result more precisely (Proofs of all results are omitted here. They are presented in [4]; Fact 1 is related, but not identical, to a result in [3]).

Fact 1. *Given the classical package,* CEM *and* SDA *imply that* $\mathsf{A} > \mathsf{C} \dashv\vdash \neg\mathsf{A} \vee C$.

Previous work on SDA has shown that it sits in major tension with the substitution of logical equivalents ([9], [8]). Interestingly, our own result makes no use of this principle. More generally, we assume nothing about the semantic or logical properties of *or*, except that it supports SDA.

3.2 Interconnectedness of all things

Our second result is that combining CEM and SDA forces the conditional to validate an undesirable schema, which we call IAT for "the Interconnectedness of All Things".

IAT. $(\mathsf{A} > \mathsf{C} \,\&\, \mathsf{B} > \mathsf{C}) \vee (\mathsf{A} > \neg\mathsf{C} \,\&\, \mathsf{B} > \neg\mathsf{C})$

Validating IAT is undesirable because it requires an extreme level of dependence among arbitrary distinct sentences. Suppose, for instance, that A="Abe flies", B="Bea runs" and C="Cleo swims". Then it must be that either both *Abe flies* > *Cleo Swims* and *Bea runs* > *Cleo swims* are true or both *Abe flies* > *Cleo does not swim* and *Bea runs* > *Cleo does not swim* are. Among other things, this appears to entail that it is incoherent to reject both of the following:

(12) If Abe flies, then Cleo swims.

(13) If Bea runs, then Cleo does not swim.

[5] For contemporary sources on the sort of system we presuppose, see [29] and [23].

It would be incorrect to say that no conditional validates IAT. For one thing, the material conditional does. Nonetheless, we comfortably assert that only unsatisfactory conditional connectives satisfy IAT. Here is an explicit statement of the second incompatibility result.

Fact 2. *Given disjunction rules, cut,* CEM*, and* SDA*,* IAT *must be a logical truth.*

3.3 *Might* conditionals

We end this section by noting a third result which, though slightly different in spirit, plays an important role in our theoretical discussion. Alonso-Ovalle [1] observes simplification with *might* conditionals (specifically counterfactuals), as in the inference from (14) to (15).

(14) If Hiro or Ezra had come, we might have solved the puzzle.

(15) If Hiro had come, we might have solved the puzzle.

Additionally, he shows that strict accounts of counterfactuals cannot validate this form of simplification, given a Boolean semantics for disjunction.

It will be convenient for our purposes to take *If A, might B* as idiomatic. Formally, we write this as $\mathsf{A} >_{\diamond} \mathsf{B}$. With this symbol in hand we state:

$\diamond$-SDA. $(\mathsf{A} \text{ or } \mathsf{B}) >_{\diamond} \mathsf{C} \vdash (A >_{\diamond} C) \,\&\, (B >_{\diamond} C)$

Note that, because we do not derive $>_{\diamond}$ compositionally, $\diamond$-SDA is not simply a special case of SDA. Nonetheless, $\diamond$-SDA is very much in the spirit of SDA itself, and plausibly supported by many of the same intuitive considerations that support SDA.

Semantically, we assume that *might*-counterfactuals existentially quantify over the very same domain that *would*-counterfactuals universally quantify over.

(S1) $[\![\mathsf{A} >_{\diamond} \mathsf{C}]\!] = \{w \mid R^w \cap [\![\mathsf{A}]\!] \cap [\![\mathsf{C}]\!] \neq \emptyset\}$

Surprisingly, this imposes severe constraints on the range of acceptable meanings for disjunction, ruling out the possibility that a disjunction like A or B has a set of possible worlds as its meaning.

Fact 3. *Assume (S1), the reflexivity of R and the validity of both* SDA *and* $\diamond$-SDA*. Then disjunction is not propositional.*

4 Alternatives

Given our incompatibility results, the prospects for reconciling SDA and CEM might appear bleak. We now turn to strategies for dealing with this tension. Our first attempt is inspired by the alternative semantics for conditionals developed in [1]. In alternative semantics, sentence meanings are not propositions, but instead sets of propositions (or 'alternatives'). A disjunction A or C presents both of A and C as alternatives. That is, $[\![\mathsf{A} \text{ or } \mathsf{B}]\!] = \{[\![\mathsf{A}]\!], [\![\mathsf{B}]\!]\}$. Disjunction contributes a set of propositions as its meaning. SDA can be validated by letting the conditional operate on each alternative in this set.

Our main idea is to derive the meaning of the conditional from an underlying propositional conditional operator $>$–the 'proto-conditional'–which maps a pair of propositions to a new proposition. The proto-conditional regulates the behavior of the conditional $\succ\!\!>$ when the antecedent is not an alternative. It also helps determine how $\succ\!\!>$ behaves when its antecedent denotes a non-trivial sets of alternatives. We illustrate this for the case in which $[\![\mathsf{A}]\!]$ denotes a set of propositions.

(S2) $[\![\mathsf{A}{\succ\!\!\succ}\mathsf{C}]\!] = \bigcap\big\{[\![>]\!](\mathbf{A}, [\![\mathsf{C}]\!]) \mid \mathbf{A} \in [\![\mathsf{A}]\!]\big\}$

To simplify a bit more, suppose the set of propositions in $[\![\mathsf{A}]\!]$ is $\{\mathbf{B}_1, ..., \mathbf{B}_j\}$ denoted by the sentences $\mathsf{B}_1, ..., \mathsf{B}_j$. Then $\mathsf{A} \succ\!\!\succ \mathsf{C}$ is true just in case each of the conditionals $(\mathsf{B}_1 > \mathsf{C}), ..., (\mathsf{B}_j > \mathsf{C})$ is true. In other words, the alternative sensitive conditional is a generalized conjunction of a series of protoconditionals, distributed over the antecedent alternatives.[6] To recycle one of our early examples, the truth-conditions of *Hiro or Ezra* $\succ\!\!\succ$ *puzzle* demand the truth of both: *Hiro* $>$ *puzzle* and *Ezra* $>$ *puzzle*.

Before showing how this framework can engage our collapse results, we must make some bookkeeping adjustments. Once we access the higher type of sets of propositions, we need a route connecting them back with propositional meanings. Without such a route, we would not be able to make sense of logical consequence. Furthermore, and relatedly, (S2) does not provide for non-disjunctive antecedents without such a bridge.

We address this problem in a somewhat non-canonical way (for the canonical approach, see [17]). Start by defining the conditional operator polymorphically. That is, let $\succ\!\!\succ$ either take a proposition or a set of propositions as input. When it takes a proposition as input, it applies $>$; otherwise, it universally quantifies over alternatives.

(S3) $[\![\mathsf{A} \succ\!\!\succ \mathsf{C}]\!] = \begin{cases} [\![\mathsf{A} > \mathsf{C}]\!] & \text{if } [\![\mathsf{A}]\!] \subseteq W \\ \bigcap\big\{[\![>]\!](\mathbf{A}, [\![\mathsf{C}]\!]) \mid \mathbf{A} \in [\![\mathsf{A}]\!]\big\} & \text{otherwise.} \end{cases}$

Next, we invoke an explicit existential closure operator !. Just like the conditional, we can define our closure operator polymorphically. When $[\![\mathsf{A}]\!]$ is a proposition, ! has no effect on A. But when $[\![\mathsf{A}]\!]$ is a set of propositions, ! takes the union of all of the A alternatives.

(S4) $[\![!\mathsf{A}]\!] = \begin{cases} [\![\mathsf{A}]\!] & \text{if } [\![\mathsf{A}]\!] \subseteq W \\ \bigcup[\![\mathsf{A}]\!] & \text{otherwise.} \end{cases}$

Then an argument is valid just in case the closure of the conclusion is true whenever the closure of all the premises are true.

(S5) $\mathsf{A}_1, \ldots, \mathsf{A}_n \models \mathsf{C}$ iff $\bigcap_{i\in[1,n]}[\![!\mathsf{A}_i]\!] \subseteq [\![!\mathsf{C}]\!]$

This proposal guarantees that disjunction behaves as classically as possible. Since entailment is only sensitive to the closed form of a sentence, we know that *or* satisfies both disjunction introduction and proof by cases.

In this framework, $[\![(\mathsf{A} \text{ or } \mathsf{B}) \succ\!\!\succ \mathsf{C}]\!] = [\![\mathsf{A} > \mathsf{C}]\!] \cap [\![\mathsf{B} > \mathsf{C}]\!]$, regardless of what $>$ means. This evidently guarantees that SDA is valid. Whether CEM is valid depends on the choice of proto-conditional $>$. Suppose, following [26], we interpret $>$ in terms of a selection function f that, given a world w and proposition $\mathbf{A}$, returns the unique closest world to w where $\mathbf{A}$ holds.

$$[\![\mathsf{A} > \mathsf{C}]\!] = \{w \mid f(w, [\![\mathsf{A}]\!]) \in [\![\mathsf{C}]\!]\}$$

Then CEM is valid for $\succ\!\!\succ$ when the antecedent is not disjunctive.[7] Furthermore, it is a simple corollary of our negative results that there is no non-trivial choice of proto-conditional that

[6] For an implementation of the same idea in inquisitive semantics, with a similar purpose to the one we have here, see [6] and [7].

[7] $\models (\mathsf{A} \succ\!\!\succ \mathsf{C}) \vee (\mathsf{A} \succ\!\!\succ \neg\mathsf{C})$ iff $[\![![(\mathsf{A} \succ\!\!\succ \mathsf{C}) \vee (\mathsf{A} \succ\!\!\succ \neg\mathsf{C})]]\!] \subseteq W$. But $[\![(\mathsf{A} \succ\!\!\succ \mathsf{C}) \vee (\mathsf{A} \succ\!\!\succ \neg\mathsf{C})]\!]$ is the set containing $[\![\mathsf{A} > \mathsf{C}]\!]$ and $[\![\mathsf{A} > \neg\mathsf{C}]\!]$, so its closure is the set of worlds where one of these conditionals holds. Since either C or $\neg\mathsf{C}$ is guaranteed to hold at $f(w, [\![\mathsf{A}]\!])$, this last is guaranteed.

validates CEM for disjunctive antecedents. Specifically, CEM fails whenever some alternatives guarantee C and some guarantee $\neg$C.

We summarize the two signature properties of the semantics above in a single statement.

Fact 4. *For any operator* $>$, $(A \textit{ or } B) \gg C \models (A \gg C)\ \&\ (A \gg C)$.
For any operator $>$, *if* $>$ *validates* CEM, *then* $\gg$ *validates* CEM *for any* A *not containing or.*

This approach dodges our first two results because those rely on applying CEM to a disjunctive antecedent, and then applying simplification. By blocking CEM for disjunctive antecedents, both proofs are blocked. The current proposal embodies a conservative response to our collapse result: it validates exactly the instances of CEM that do not lead to trouble when combined with SDA.

The problem, however, is that the motivation for CEM does not appear to discriminate against disjunctive antecedents. For instance, (16) and (17) sound equivalent in just the same way that (5) and (6) do

(16) I doubt that if you had slept in or goofed off, you would have passed.

(17) I believe that if you had slept in or goofed off, you would have failed.

Similarly, we observe a duality effect with disjunctive antecedents under *no* and *every*. As before, (18) and (19) appear equivalent.

(18) No student would have succeeded if he had goofed off in class or partied the night before the exam.

(19) Every student would have failed if he had goofed off in class or partied the night before the exam.

By restricting CEM, the analysis renounces these predictions.

Turning to *only if*, we saw that CEM is quite useful in deriving the meaning of *only if* conditionals compositionally from the interaction of *only* and conditionals. Our question now is whether *only if* conditionals with disjunctive antecedents imply their converses.

(20) The flag flies only if the King or Queen is home.

(21) If the flag flies, then the King or Queen is home.

(22) The flag flies if the King or Queen isn't home.

It is clear that (20) does imply (21), just as we saw earlier that (9) implied (10). This is a problem for the analysis above, which denies CEM for conditionals with disjunctive antecedents. For, again, a natural way to predict this entailment is through the idea that *only* negates alternatives, and that (22) is an alternative to the conditional in (20). But if CEM fails for disjunctive antecedents, then the negation of (22) will not imply the contraposition of (21), which is essential in [11]'s account.

Summing up: with alternative semantics, we can enforce SDA while restricting the validity of CEM to non-disjunctive antecedents. However, this restriction is not justified in light of the justification of CEM. For this reason, we now turn to another strategy for avoiding collapse.

5 Homogeneity

We might approach things from the opposite angle: instead of taking an arbitrary conditional and forcing the validity of SDA, we might force the validity of CEM.

5.1 Homogeneity presuppositions

The instrument that yields this result is the theory of homogeneity presuppositions. Homogeneity presuppositions have been invoked to explain certain otherwise problematic variants of excluded middle for plural definites (see for example [11]). In that context, the problem starts with the observation that predications involving plural definites, like (23), plausibly license inferences to universal claims like (24).

(23) The cherries in my yard are ripe.

(24) All the cherries in my yard are ripe.

If some but not all cherries are ripe, one would not be in a position to assert (23). Furthermore, plural definites plausibly exclude the middle. That is, the following sounds like a logical truth:

(25) Either the cherries in my yard are ripe or they (=the cherries in my yard) are not ripe.

If someone were to utter (25), they would sound just about as informative as if they had made a tautological statement (although you might learn from it that they have cherries in their yard). The problem is that, starting with (25) and exploiting entailments like the one from (23) to (24) as well as standard validities for disjunction, we can reason our way to (26):

(26) Either all the cherries in my yard are ripe or all the cherries in my yard are not ripe.

That seems puzzling: did we just prove from logical truths and valid inferences that my yard cannot have some ripe cherries and some non-ripe ones? Of course, something must have gone wrong. The homogeneity view of plural definites explains what that is: first, plural definites carry a presupposition of homogeneity: *the F's are G's* presupposes that the F's are either all G's or all not G's. If this presupposition is satisfied, their content is that all F's are G's. The sense in which (25) sounds tautological is that it cannot be false if its homogeneity presupposition is satisfied. Similarly, the sense in which (23) entails (24) is that if the presupposition of (23) is satisfied and (23) is true, (24) cannot fail to be true. But even if we exploit these to deduce (26), we do not have license us to claim that (26) is valid: our reasoning did not discharge the homogeneity presupposition.

5.2 Forcing CEM via homogeneity

A treatment of CEM using homogeneity presuppositions [11] allows that there may be more than one relevant world where the antecedent of a conditional is true. The key idea is that $\mathsf{A} > \mathsf{C}$ presupposes that C is true at all of the relevant worlds where A is true, or false at all of them. The A-worlds must be "homogeneous" with respect to the consequent.

We generalize the proposal of [11] by reformulating the theory without any appeal to quantification over worlds. Instead, we take an arbitrary conditional operator $>$, and enrich it with homogeneity presuppositions to create a new conditional, $\cdot\!\cdot\!>$.

(S6) $[\![\mathsf{A} \cdot\!\cdot\!> \mathsf{C}]\!](w)$ is defined only if $[\![\mathsf{A} > \mathsf{C}]\!](w) = 1$ or $[\![\mathsf{A} > \neg\mathsf{C}]\!](w) = 1$.
If defined, $[\![\mathsf{A} \cdot\!\cdot\!> \mathsf{C}]\!](w) = [\![\mathsf{A} > \mathsf{C}]\!](w)$.

To talk about SDA and CEM, we also need appropriate assumptions about $\neg$ and $\vee$. These connectives must allow homogeneity presuppositions to project in the right way. To this end, we assume that $[\![\neg\mathsf{A}]\!](w)$ is defined only if $[\![\mathsf{A}]\!](w)$ is defined; if defined, $[\![\neg\mathsf{A}]\!](w) = 1 - [\![\mathsf{A}]\!](w)$.

As for disjunction we assume that $[\![\mathsf{A} \vee \mathsf{B}]\!](w)$ is defined only if $[\![\mathsf{A}]\!](w)$ and $[\![\mathsf{B}]\!](w)$ are defined; if defined, $[\![\mathsf{A} \vee \mathsf{B}]\!](w) = max([\![\mathsf{A}]\!](w), [\![\mathsf{B}]\!](w))$.

Finally, to get predictions about our collapse results, we need a definition of consequence. The leading candidate for languages involving presuppositions is Strawson-validity [28], [11], [12], [13]. According to this notion, an argument is valid just in case the conclusion is true whenever the conclusion is defined and the premises are true.

(S7) $\mathsf{A}_1; \ldots ; \mathsf{A}_n \models \mathsf{C}$ iff $[\![\mathsf{C}]\!](w) = 1$ whenever:

- $[\![\mathsf{A}_1]\!](w); \ldots ; [\![\mathsf{A}_n]\!](w)$ are defined.
- $[\![\mathsf{A}_1]\!](w) = 1$ and $\ldots$ and $[\![\mathsf{A}_n]\!](w) = 1$.
- $[\![\mathsf{C}]\!](w)$ is defined.

The first important result is that CEM is valid regardless of the choice of proto-conditional. The key result, however, is that any proto-conditional $>$ that validates SDA induces a new conditional $\cdots\!>$ that also validates SDA. Indeed, this is not unique to simplification.

Fact 5. *(i) For any operator $>$, $\models (A \cdots\!> C) \vee (A \cdots\!> \neg C)$; (ii) For any operator $>$, if $>$ validates* SDA*, then $\cdots\!>$ validates* SDA*.*

We now have a completely general recipe for validating both SDA and CEM. But have we avoided the bad consequences we claimed should follow? For example, is it the case that for any operator $>$ that validates SDA, $\cdots\!>$ collapses to the material conditional? The answer to both questions is "no". There are many choices of protoconditional for which $\cdots\!>$ is not trivial. A first example is if we let $>$ be a generic strict conditional. To see how this theory avoids triviality, let us look at the semantic correlates of some of the entailments we used in the proof of our first collapse result. The first step of the proof of Fact 1 corresponds to this semantic fact: (27) is a logical truth.

(27) $[(\mathsf{A} \vee \neg \mathsf{A}) \cdots\!> \mathsf{C}] \vee [(\mathsf{A} \vee \neg \mathsf{A}) \cdots\!> \neg \mathsf{C}]$

Although (27) is true whenever defined, it is quite difficult for it to be defined. Given our account of $\vee$, the definedness of $(\mathsf{A} \vee \neg \mathsf{A}) > \mathsf{C}$ is equivalent to the requirement that either $R^w \subseteq [\![\mathsf{C}]\!]$ or $R^w \subseteq [\![\neg \mathsf{C}]\!]$. One of C and $\neg \mathsf{C}$ must be necessary at w (in the relevant sense of necessity) for (27) to be defined.

Now, the reasoning connecting the first two steps of our proof also has a matching semantic fact: (28) entails (29).

(28) $[(\mathsf{A} \vee \neg \mathsf{A}) \cdots\!> \mathsf{C}] \vee [(\mathsf{A} \vee \neg \mathsf{A}) \cdots\!> \neg \mathsf{C}]$

(29) $[(\mathsf{A} \cdots\!> \mathsf{C} \ \& \ \neg \mathsf{A} \cdots\!> \mathsf{C}) \vee [(\mathsf{A} \cdots\!> \neg \mathsf{C} \ \& \ \neg \mathsf{A} \cdots\!> \neg \mathsf{C})]$

This holds because if (27) is defined, then the domain R^w uniformly consists of C-worlds or it uniformly consists of $\neg\mathsf{C}$-worlds. Either way, (29) must be true.

Despite the validity of (27) and the entailment from (27) to (29), (29) is not itself valid. The definedness conditions of (29) are laxer than those of (27): for this reason (29) has a much better shot at being false. For instance (29) is false in a model that contains two worlds w and v with w verifying A and C and v verifying $\neg\mathsf{A}$ and $\neg\mathsf{C}$. But such a model does not impugn the validity of (27) under Strawson entailment, because its disjuncts are undefined.

In broad strokes, an instance of transitivity—in particular, one of the form $\models \mathsf{A}$, $\mathsf{A} \models \mathsf{B}$, therefore $\models \mathsf{B}$—fails for Strawson entailment [25]. This is possible because $\models \mathsf{A}$ only requires that A be true if defined; meanwhile, $\mathsf{A} \models \mathsf{B}$ also holds because the presuppositions of A are essentially involved in guaranteeing the truth of B. But $\models$ B fails because here we are not allowed to assume that the presuppositions of A are satisfied. The same diagnosis applies to our second impossibility result. The first step of the proof claims the validity of $[(\mathsf{A}$ or $\mathsf{B}) > C] \vee [(\mathsf{A}$ or $\mathsf{B}) > \neg\mathsf{C}]$. The argument establishes that this claim entails IAT. However, the validity of IAT does not follow for a parallel reason to the one we uncovered in discussing the first result.

6 Synthesis

We argued that a generic strict conditional $>$ can validate both SDA and CEM, when enriched with homogeneity presuppositions. Here, however, we must take care. The resulting theory validates SDA, but invalidates $\Diamond$-SDA. That is, the analogue of simplification of disjunctive antecedents for *if ... might ...* fails to be preserved. This is a problem because $\Diamond$-SDA sounds no less plausible than SDA itself.

To fully validate simplification, we propose a synthesis of our two tools. In particular, we suggest that the English conditional recruits *both* alternatives and homogeneity presupposition. To signal this fact, we now introduce the new connective $\cdot\!\cdot\!\succ\!\succ$. Start with any conditional meaning. Then apply the alternative sensitive enrichment from (S5). The resulting semantics validates both SDA and $\Diamond$-SDA, but invalidates CEM for disjunctive antecedents. To validate CEM unrestrictedly, enrich this conditional with homogeneity presuppositions.

More precisely, given an arbitrary proto-conditional $>$, we characterize $\cdot\!\cdot\!\succ\!\succ$ by the clauses:

(S8a) If $[\![\mathsf{A}]\!] \subseteq W$, then $[\![\mathsf{A} \cdot\!\cdot\!\succ\!\succ \mathsf{C}]\!](w)$ is defined only if $[\![\mathsf{A}{>}\mathsf{C}]\!](w) = 1$ or $[\![\mathsf{A} > \neg\mathsf{C}]\!](w) = 1$.
If defined, $[\![\mathsf{A} \cdot\!\cdot\!\succ\!\succ \mathsf{C}]\!] = [\![\mathsf{A}\succ\!\succ \mathsf{C}]\!] = [\![\mathsf{A}{>}\, \mathsf{C}]\!]$.

(S8b) Otherwise, $[\![\mathsf{A} \cdot\!\cdot\!\succ\!\succ \mathsf{C}]\!](w)$ is defined only if either $[\![>]\!](\mathbf{A})([\![\mathsf{C}]\!])(w) = 1$ for every $\mathbf{A} \in [\![\mathsf{A}]\!]$, or $[\![>]\!](\mathbf{A})([\![\mathsf{C}]\!])(w) = 0$ for every $\mathbf{A} \in [\![\mathsf{A}]\!]$.
If defined, $[\![\mathsf{A} \cdot\!\cdot\!\succ\!\succ \mathsf{C}]\!] = [\![\mathsf{A}\succ\!\succ \mathsf{C}]\!] = \bigcap\{[\![>]\!](\mathbf{A})([\![\mathsf{C}]\!]) \mid \mathbf{A} \in [\![\mathsf{A}]\!]\}$.

Crucially, there are choices of proto-conditional for which the recipe does not yield a collapsing conditional. In particular, a natural option for the proto-conditional is the Lewisian variably strict conditional. The underlying Lewisian operator allows that there may be multiple worlds where the antecedent is true that are relevant to the evaluation of the consequent. Then the conditional that results from applying the procedure above is doubly homogenous. First, the conditional presupposes that the antecedent alternatives either all guarantee the consequent, or all guarantee the consequent's negation. Second, for each antecedent alternative, the conditional presupposes that either all of the relevant worlds where that alternative holds are worlds where the consequent is true, or they are all worlds where the consequent is false. Perhaps surprisingly, this theory more or less has already been developed and endorsed, for somewhat different reasons, in [24].

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